HOMEWORK FOR MATH 147 SPRING 2021

Problems indicated by section number refer to our main text Apex Calculus II by G. Hartman. Problems assigned from Marsden and Weinstein will be preceded by MW. For problems assigned from either textbook, turn in the **boldface** numbers only. Problems assigned from the Openstax calculus 3 textbook will be preceded by OS.

Week of February 1. Read sections 12.2 and 12.2 in Hartman and

(i) Experiment with graphing functions of two variables with the online graphing function Desmos. Turn in a couple of pictures of interesting surfaces you created.
(ii) Section 12.1: 11, 13, 15, 20, 21, 23, **14, 17, 22**.
(iii) Section 12.2: 10, 12, 15, 17, 21, **11, 14, 18, 20**.
(iv) Turn in solutions to the following problems:

(a) For $\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+2y^2}$, show that the limit along any line through the origin exists and equals zero, but if we take the limit along the curve $y = x^3$, the limit is not zero. What conclusion can you draw from this?

(b) Determine if the following limit exists, and if so, what is its value: $\lim_{(x,y)\to(0,0)} \frac{x^5+y^4-3x^3y+2x^2+2y^2}{x^2+y^2}$.

(c) Determine if the function $f(x,y) = \begin{cases} \frac{x^3+x^2+xy^2+y^2}{x^2+y^2}, & (x,y) \neq (0,0) \\ 2, & (x,y) = (0,0) \end{cases}$ is continuous at (0,0).

(d) At what values of (x, y) is the function $f(x,y) = \frac{\cos(x^2-y^2)}{x^2+1}$ continuous?

Optional Bonus Problem. 3points. Use items (i) and (ii) below to show that for $f(x,y) = xy\frac{x^2-y^2}{x^2+y^2}$, $\lim_{(x,y)\to(0,0)} f(x,y)$ exists, and then find its value.

(i) Prove that if a, b are real numbers, then $2|ab| \leq a^2 + b^2$.
(ii) Use (i) to show that if $0 < |(x,y)| < \delta$, then $|f(x,y)| < \frac{\delta^2}{2}$.

This assignment is due on Blackboard by 11:59pm on Monday, February 8.

Week of February 8. Read sections 15.1 and 15.2 in Marsden and Weinstein and section 12.6 in Hartman and

(i) MW section 15.1: 9-41, odd; **8**, **7**, **15**, **33**, **41**, **75**.
(ii) MW section 15.2: 5, **7**, 9, 26, **27**.
(iii) Section 12.6: 13, **15**, 17, **19**, 21, 23, **27**.
(iv) Turn in solutions to the following problems:

(a) For $f(x,y) = x^2y + xy^2$, use the limit definitions to find $f_x(2,3)$ and $f_y(2,3)$.

(b) For $f(x,y) = 3x^2 + 2xy + 4$, use the limit definition to find the directional derivative of $f(x,y)$ at (1,1) in the direction of the vector $a\vec{i} + b\vec{j}$.

(c) For $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, \text{if}(x,y) \neq (0,0) \\ 0, \text{if}(x,y) = (0,0) \end{cases}$, determine if $f_x(x,y)$ is continuous at (0,0).

This assignment is due on Blackboard by 11:59pm on Monday, February 15.

Week of February 15. Read sections 15.3 and 15.4 in Marsden and Weinstein and Section 12.5 in Hartman. And work the following problems:

(i) MW, 15.1, **71, 72**
(ii) MW 15.2, 17-27, **17, 22, 27**
(iii) MW 15.3, 9-21, odd, **9, 11, 22**

(iv) MW 15.4, **25, 26, 29**

Also, turn in the following problems:

(a) Use the limit definition to prove that $f(x,y) = x^2y + xy^2$ is differentiable at (1,1).

(b) For the function $f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$

(i) Determine if $f(x,y)$ is continuous at (0,0).
(ii) Determine if $f_x(0,0)$ and $f_y(0,0)$ exist, and if so, what are their values.
(iii) Show that $f(x,y)$ is not differentiable at (0,0).
(iv) Explain, with justification, why $f(x,y)$ fails to differentiable at (0,0).

This assignment is due on Blackboard by 11:59pm on Monday, February 22.

Week of February 22. Read Section 12.8 in Hartman and Section 16.3 in Marsden and Weinstein and work the following problems:

(i) Section 12.8, 1-17, odd, and **9, 12, 13, 15, 17**.
(ii) MW, Section 16.3, **24, 28, 32, 35**.

Optional Bonus Problem. Read the prelude to problems 43-48 in MW Section 16.3. Then use the techniques of this homework section to find the equation of the line of best fit passing through data points $(x_1, y_1), \ldots, (x_n, y_n)$.

This assignment is due on Blackboard by 11:59pm on Monday, March 1.

Week of March 1. Read section 16.4 in Marsden and Weinstein and wrk the following problems.

(i) MW, Section 16.4, 2-19, odd, **7, 9, 11, 19, 20**.
(ii) Hartman, Section 12.8, **15, 17**.
(iii) Turn in the following problems:
  (a) Find and classify the critical points of $f(x,y,z) = x^3 + xz^2 - 3x^2 + y^2 + 2z^2$ using the second derivative test for functions of three variables.
  (b) Use Lagrange multipliers to find and classify the extreme values $f(x,y,z) = x^2+y^2+z^2$, subject to $x + y - z = 1$.
  (c) Use Lagrange multipliers to find and classify the extreme values of $f(x,y,z) = x^6 + y^6 + z^6$, subject to $x^2 + y^2 + z^2 = 6$.

Optional Bonus Problem. This problem fills in some of the details of our discussion concerning the second derivative test for functions of two variables. Assume we are given $f(x,y)$ with continuous second order partial derivatives, and $f_x(a,b) = 0 = f_y(a,b)$. Thus, from our discussion in class, we have

$$f(a+h_1, b+h_2) \approx f(a,b) + \frac{1}{2} \cdot \tilde{Q}(h_1,h_2),$$

if h_1, h_2 are sufficiently small, where $\tilde{Q}(h_1,h_2) = f_{xx}(a,b)h_1^2 + 2f_{xy}(a,b)h_1h_2 + f_{yy}(a,b)h_2^2$. Under these conditions, show the following.

(i) If $\tilde{Q}(h_2,h_2) > 0$, for all (h_1,h_2), then $f(x,y)$ has a relative minimum value at (a,b). (2.5 points)
(ii) Suppose $P(x) = Ax^2 + 2Bx + C$, $A > 0$ and $AC - B^2 > 0$. Then $P(x) > 0$, for all x. (2.5 points). Hint: Does the graph of $P(x)$ cross the x-axis?
(iii) For A, B, C as in (ii), conclude that $Ah_1^2 + 2Bh_1h_2 + Ch_2^2 > 0$, for all non-zero h_1, h_2. (2.5 points)
(iv) For $f(x,y)$ as above, if $f_{xx}(a,b) > 0$ and $D(a,b) > 0$, then $f(x,y)$ has a relative minimum at (a,b). (2.5 points)

This assignment is due on Blackboard by 11:59pm on Monday, March 8.

Week of March 8. Read Section 13.2 in Hartman and Section 17.3 in Marsden and Weinstein. Work the following:

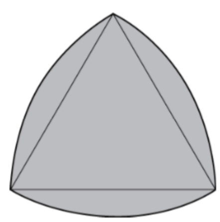
(i) Hartman, Section 13.1, 14-16, **15**.
(ii) Hartman, Section 13.2, 5-25, odd, **7, 11, 15, 17, 19, 21, 25**.
(iii) MW, Section 17.2, **17, 20**.
(iv) MW, Section 17.3, **5, 10, 17, 23**.
(v) Find the surface area of the closed cone $z^2 = x^2 + y^2$, with $0 \le z \le 4$ (to be turned in).

This assignment is due on Blackboard by 11:59pm on Monday, March 15.

Week of March 15. In OpenStax calculus 3, read the last portion of Section 5.2 pertaining to improper double integrals, and that portion of section 5.7 referring to change of variables for double integrals. Work the following problems:

(i) OS, Section 5.3, 153, **159**, 161, 164, **173, 175, 178**.
(ii) OS, Section 5.2, 103, **105, 113**.
(iii) OS, Section 5.7, 363, **365**, 369, **371, 389, 391, 393, 395**.
(iv) And the following problems to be turned in
  (a) Use double integration to determine the volume of an inverted cone whose height is h and whose circular base has radius R.
  (b) $\int\int_b \frac{1}{\sqrt{xy}}\ dA$, where $D = [0,1] \times [0,1]$. Hint: For $0 < a, b < 1$, consider the integrals $\int\int_{D_{a,b}} \frac{1}{\sqrt{xy}}\ dA$ and take a limit as $a, b \to 0$, where $D_{a,b} = [a,1] \times [b,1]$.
  (c) $\int\int_D \frac{1}{x^2y^3}\ dA$, where $D = [1,\infty) \times [1,\infty)$.
  (d) $\int\int_{\mathbb{R}^2} \frac{1}{(1+x^2+y^2)^p}\ dA$, for $p > 1$.

Optional Bonus Problems. (a) The Reuleaux triangle consists of an equilateral triangle and three regions, each of them bounded by a side of the triangle and an arc of a circle of radius s centered at the opposite vertex of the triangle. Show that the area of the Reuleaux triangle in the following figure of side length s equals, $\frac{s^2}{2}(\pi - \sqrt{3})$. (5 points)



(b) Find the volume between the surfaces $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$ over the domain $D : 0 \le x^2 + y^2 \le 16$. (3 points)

This assignment is due on Blackboard by 11:59pm, Monday, March 22.

Week of March 29. Read OS, sections 5.4 and 5.5 and work the following problems. OS, Chapter 5: 183, **185**,189, 191, **193**, 195, **197**, 203, **205, 207, 211, 219, 223, 233**, 241, **243**, 245, **247**, 249, **253**, 269, **271, 275, 281, 283**.

This assignment is due on Blackboard by 11:59pm, Tuesday April 6.

Week of April 5. Read OS, Section 5.7 concerning the change of variables formula for triple integrals and work the following problems:

(i) MW, section 17.5, 7-16, **7, 9, 11, 13, 15, 18**.
(ii) OS, Chapter 5: **399, 401, 405, 413** .
(iii) MW, section 14.6: **17, 19, 35, 45, 51**.

Optional Bonus Problems. In class we talked about improper double integrals. First, explain why the triple integrals below are improper and then extend the techniques from class to work the improper triple integrals.

1. $\int\int\int_B \ln\sqrt{x^2+y^2+z^2}\ dV$, where B is the solid sphere of radius R centered at the origin. (2.5 points)

2. $\int\int\int_{\mathbb{R}^3} e^{-\sqrt{x^2+y^2+z^2}}\ dV$. (2.5 points)

This assignment is due on Blackboard by 11:59pm, Monday April 12.

Week of April 12. Read MW section 18.1 and OS Section 6.6 and work the following problems:

(i) OS, Chapter 3: 103, 104, **105, 107**, 110, **111**.
(ii) MW, Section 18.1: **29, 20, 31, 32, 33, 34**.
(iii) OS, Chapter 6: 272, **275, 277**, 278, **281**, 282, **283**, **309**, **311**, **313**.

This assignment is due on Blackboard by 11:59pm, Tuesday April 20.

Week of April 19. Work the following problems.

(i) MW, Section 18.1, 13-20, 27, **17, 19, 21, 25, 28**.
(ii) MW, Section 18.5, **5**, 6, **7, 23**.
(iii) OS, Chapter 6, **302, 303, 304, 305, 315, 316, 317**.

This assignment is due on Blackboard by 11:59, Monday, April 26.

Week of April 26. Work the practice problems for Exam 3.

This assignment is due on Blackboard by 11:59, Monday May 1.